THE UNIVERSITY OF ADELAIDE
DEPARTMENT OF MECHANICAL ENGINEERING

EXAMINATION FOR THE DEGREE OF B.E.

2391: DYNAMICS

FINAL EXAMINATION - NOVEMBER, 2002

TIME: 2 HOURS

Number of pages	**8**
Number of Questions	**4**
Total Marks	**76**

[In addition the candidates are allowed ten minutes before the exam to read the paper.]

[Notes and textbooks are **not** permitted in the examination room.]

[Calculating devices are permitted]

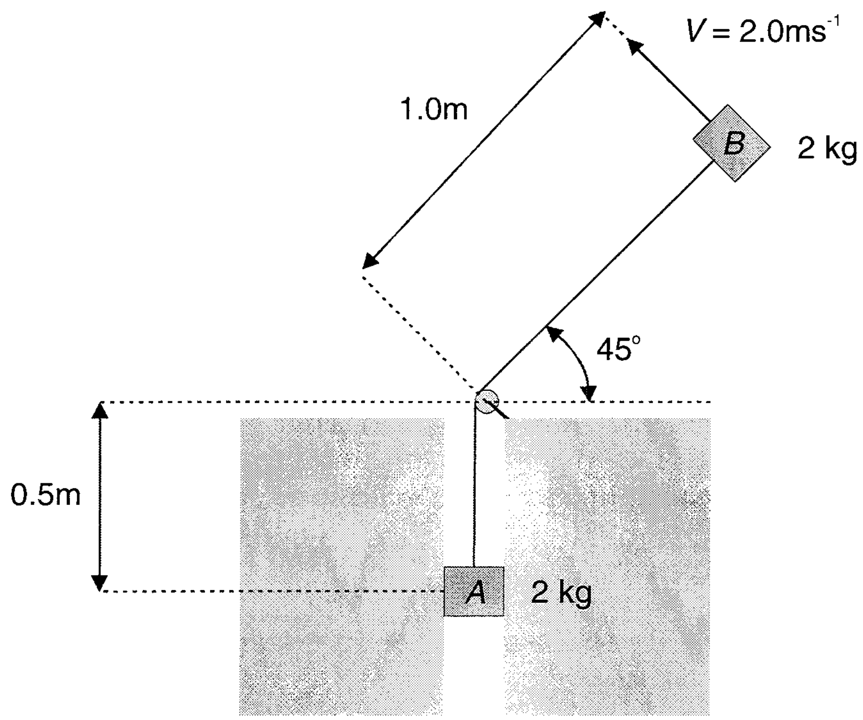
[A formula sheet is provided on the back sheet of the exam booklet (Pages 7 and 8). This may be torn off for easy reference]

Attempt ALL **FOUR** questions.

1) At the instant shown in Figure 1, mass *B* is travelling in the direction shown (perpendicular to the taut cord) with speed $v_B = 2.0ms^{-1}$.

Assume the arrangement of masses to be on a frictionless horizontal table. There is negligible friction between *A* and the blocks either side of it. The tension in the cord is constant along its entire length.

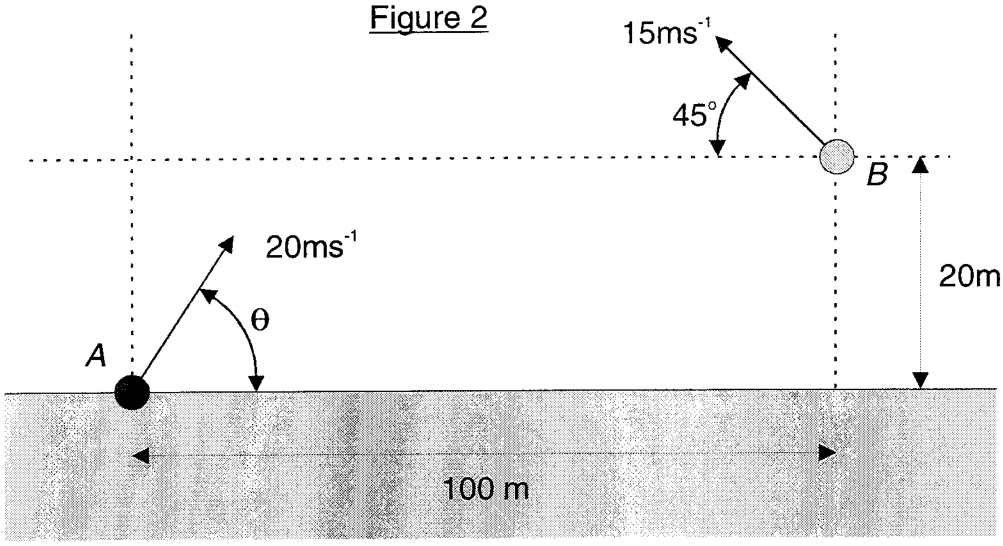
Figure 1



What is the acceleration of mass *B* at this instant? (Note: this is a kinetic problem - not just kinematics) **(14 marks)**

2) Projectile A, launched from the ground with a speed of 20.0 ms^{-1}, must strike another projectile B in flight. At the instant that A is launched, B is travelling at 15.0 ms^{-1} at an angle of 45° above the horizontal to the left, and is located 100 m to the right of A, and 20 m above A.

Figure 2



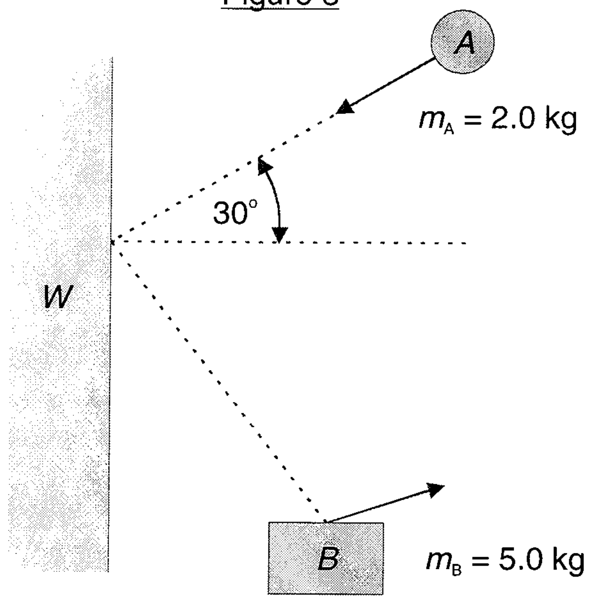
If A is launched at an angle to strike B, what is the relative velocity of A with respect to B at the moment of impact? **(22 Marks)**

You may neglect air resistance and assume that $g = 9.81$ ms^{-2} downward for both projectiles.

In order to solve for the launch angle of A, the trigonometric identity $\sin^2\theta = 1 - \cos^2\theta$ may be useful.

3) Ball *A* is initially travelling in the direction shown in the figure at a speed of 1.0 ms^{-2}, in the horizontal plane on a frictionless surface.

Figure 3

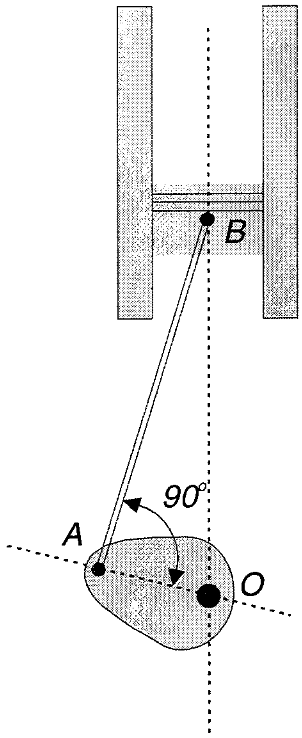


What is the final velocity of *A* after bouncing off the fixed wall *W* and then the stationary (but not fixed) block *B*? **(20 Marks)**

Assume that coefficient of restitution for both collisions is 0.8, and that the impact between *A* and *B* is central (i.e. The line of impact passes through the centres of mass of both objects).

4) At the instant shown the velocity of the piston (*B*) is 10 ms^{-1} downward with an acceleration of 4000 ms^{-2} downwards.

Figure 4



$\overline{OA} = 0.05$ m

$\overline{AB} = 0.15$ m

What is the angular acceleration of the crank shaft (*OA*) at this instant?

(20 Marks)

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PLEASE SEE NEXT PAGE

Dynamics 2391 Formula Sheet

Rectangular Co-ordinates

$\boldsymbol{r} = x\hat{\boldsymbol{i}} + y\hat{\boldsymbol{j}}$ $\quad \boldsymbol{v} = \dot{x}\hat{\boldsymbol{i}} + \dot{y}\hat{\boldsymbol{j}}$ $\quad \boldsymbol{a} = \ddot{x}\hat{\boldsymbol{i}} + \ddot{y}\hat{\boldsymbol{j}}$ (Dots indicate time derivatives)

Polar Co-ordinates

$\boldsymbol{r} = r\hat{\boldsymbol{e}}_r$ $\quad \boldsymbol{v} = \dot{r}\hat{\boldsymbol{e}}_r + r\dot{\theta}\hat{\boldsymbol{e}}_\theta$ $\quad \boldsymbol{a} = (\ddot{r} - r\dot{\theta}^2)\hat{\boldsymbol{e}}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\boldsymbol{e}}_\theta$

Tangential and Normal Co-ordinates

$$\boldsymbol{v} = v\hat{\boldsymbol{e}}_t \qquad \boldsymbol{a} = \dot{v}\hat{\boldsymbol{e}}_t + \frac{v^2}{\rho}\hat{\boldsymbol{e}}_n$$

Radius of Curvature

$$\rho = \frac{\left(1 + y'^2\right)^{3/2}}{|y''|}, \text{ where } y' = \frac{dy}{dx} \text{ and } y'' = \frac{d^2y}{dx^2}$$

Relative Motion (in *non-rotating* co-ordinates)

$\boldsymbol{r}_B = \boldsymbol{r}_A + \boldsymbol{r}_{B/A}$, $\quad \boldsymbol{v}_B = \boldsymbol{v}_A + \boldsymbol{v}_{B/A}$, $\quad \boldsymbol{a}_B = \boldsymbol{a}_A + \boldsymbol{a}_{B/A}$

Transformation of a vector between 2 co-ordinate systems rotated relative to each other.

$\boldsymbol{b} = b_x\boldsymbol{i} + b_y\boldsymbol{j} = b_{\tilde{x}}\boldsymbol{i}^{\sim} + b_{\tilde{y}}\boldsymbol{j}^{\sim}$ If the $\boldsymbol{i}^{\sim}$, $\boldsymbol{j}^{\sim}$ unit vectors are rotated an angle θ anti-clockwise with respect to *i and j* then

$$b_{\tilde{x}} = b_x\cos(\theta) + b_y\sin(\theta)$$
$$b_{\tilde{y}} = -b_x\sin(\theta) + b_y\cos(\theta)$$

Newton's Second Law for a system of *N* particles

$$\sum \boldsymbol{F}_i = m_T\,\boldsymbol{a}_G$$

where m_T is the total mass of the system, $\boldsymbol{a}_G = \ddot{\boldsymbol{r}}_G$ is the acceleration of the centre of mass, $\boldsymbol{r}_G$, which is defined by $\boldsymbol{r}_G = \left(\sum_{i=1}^{N} m_i\boldsymbol{r}_i\right) \Big/ m_T$

Newton's Law of Universal Gravitation

$$\boldsymbol{F} = \frac{Gm_1m_2}{r^2} \quad \text{where } G = 6.673\text{x}10^{-11}\ \text{m}^3\text{kg}^{-1}\text{s}^{-2}$$

Central Force Motion

$$r^2\dot{\theta} = \text{constant}$$

Kinetic Energy of a particle of mass *m* and velocity *v*

$$T = \frac{1}{2}mv^2$$

Principle of Work/Energy

$\Delta T = U_{1\to 2}$, where $U_{1\to 2}$ is the work done by forces on a system between states 1 and 2.

$U_{1\to 2} = \sum \int \boldsymbol{F} \bullet d\boldsymbol{r}$. For a conservative force, $U_{1\to 2} = -\Delta V$, where V is the potential energy of the force.

Potential Energy Functions

For a linear spring: $V = \frac{1}{2} k \delta^2$

For a mass in a uniform gravitational field: $V = mgy$

For a mass in a general gravitational field: $V = -\frac{GMm}{r}$

Power and Efficiency

$Power = \frac{dU}{dt} = \boldsymbol{F} \bullet \boldsymbol{v}$, $Efficiency = \eta = \frac{Output - Work}{Input - Work}$

Principle of Impulse and Momentum

$Impulse = \Delta \boldsymbol{p} = \int \boldsymbol{F}\, dt$

Coefficient of Restitution

$$e = \frac{v'_{Bn} - v'_{An}}{v_{An} - v_{Bn}}$$

Relative Motion of 2 points on a Rigid body

For a polar co-ordinate system attached at A,

$$\boldsymbol{v}_B = \boldsymbol{v}_A + (\boldsymbol{v}_{B/A})_\theta = \boldsymbol{v}_A + \boldsymbol{r}_{B/A} \omega_{BA} \hat{\boldsymbol{e}}_\theta$$

$$\boldsymbol{a}_B = \boldsymbol{a}_A + (\boldsymbol{a}_{B/A})_r + (\boldsymbol{a}_{B/A})_\theta = \boldsymbol{a}_A + (-\boldsymbol{r}_{B/A} \omega_{BA}^2) \hat{\boldsymbol{e}}_r + (r_{B/A} \alpha_{BA}) \hat{\boldsymbol{e}}_\theta$$

(Note: $\omega_{BA} = \omega_{AB}$ is the angular velocity of the link AB, and $\alpha_{BA} = \alpha_{AB}$ is the angular acceleration of the link AB)

Some old maths formulae you may have forgotten

For the quadratic equation, $ax^2 + bx + c = 0$, the roots are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The sine and cosine rules

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c} \qquad a^2 = b^2 + c^2 - 2bc\cos\alpha$$