THE UNIVERSITY OF ADELAIDE
DEPARTMENT OF MECHANICAL ENGINEERING

EXAMINATION FOR THE DEGREE OF B.E.

2391: DYNAMICS

NOVEMBER, 2001

TIME: 2 HOURS & 10 MINUTES

[Students are advised to devote 10 minutes to reading the paper and planning their approach.]

[Notes and textbooks are **not** permitted in the examination room.]

[Calculating devices are permitted.]

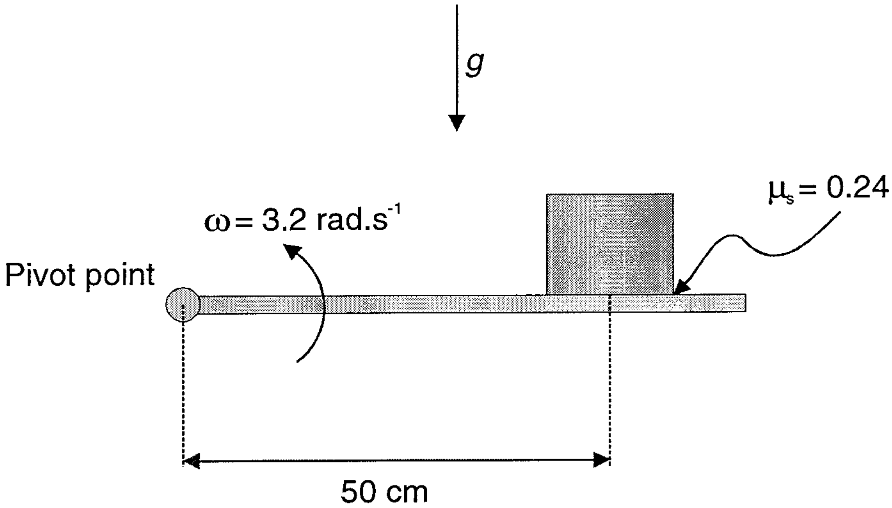
[The Dynamics formula sheet is provided at the end of the exam paper.]

[Total Marks : 84]

Attempt ALL **FOUR** questions.

1) A block sits on a rotating arm a distance of 50cm from the centre of rotation. At $t = 0$ the arm is in the horizontal position as shown in Figure 1. If the arm rotates at a constant angular velocity of 3.2 rad.s^{-1} in an anti-clockwise direction, and the static coefficient of friction between the block and arm is 0.24, how long will it take before the block slides? **(20 Marks)**

Figure 1



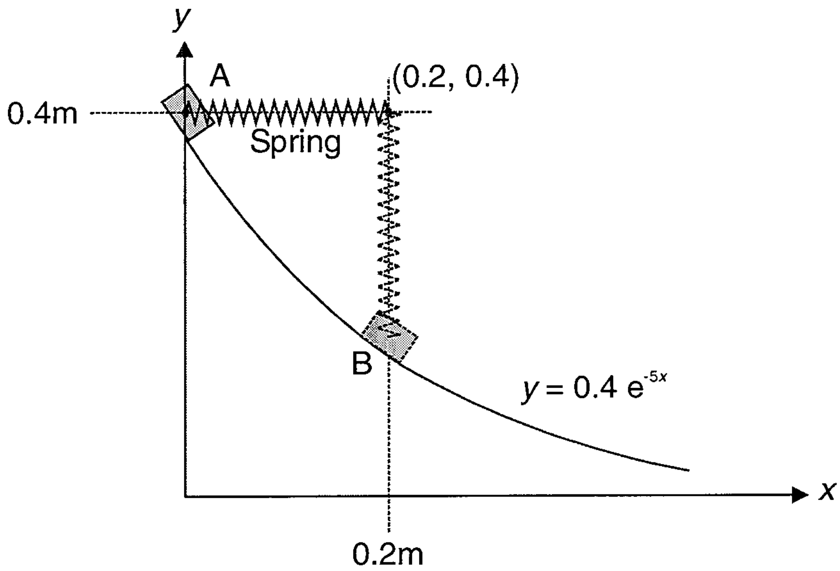
Note: The gravitational force acts downward.

Hint: To solve an equation of the form $a\cos x + b\sin x = c$, use the substitution $\cos^2 x = 1 - \sin^2 x$

2) Consider Figure 2. The apparatus shown is aligned in a vertical plane on the Earth's surface. The mass is released from rest from position A as shown. It proceeds to slide down along the curved frictionless surface described by the formula

$y = 0.4\,e^{-5x}$ where x and y are in metres.

Figure 2



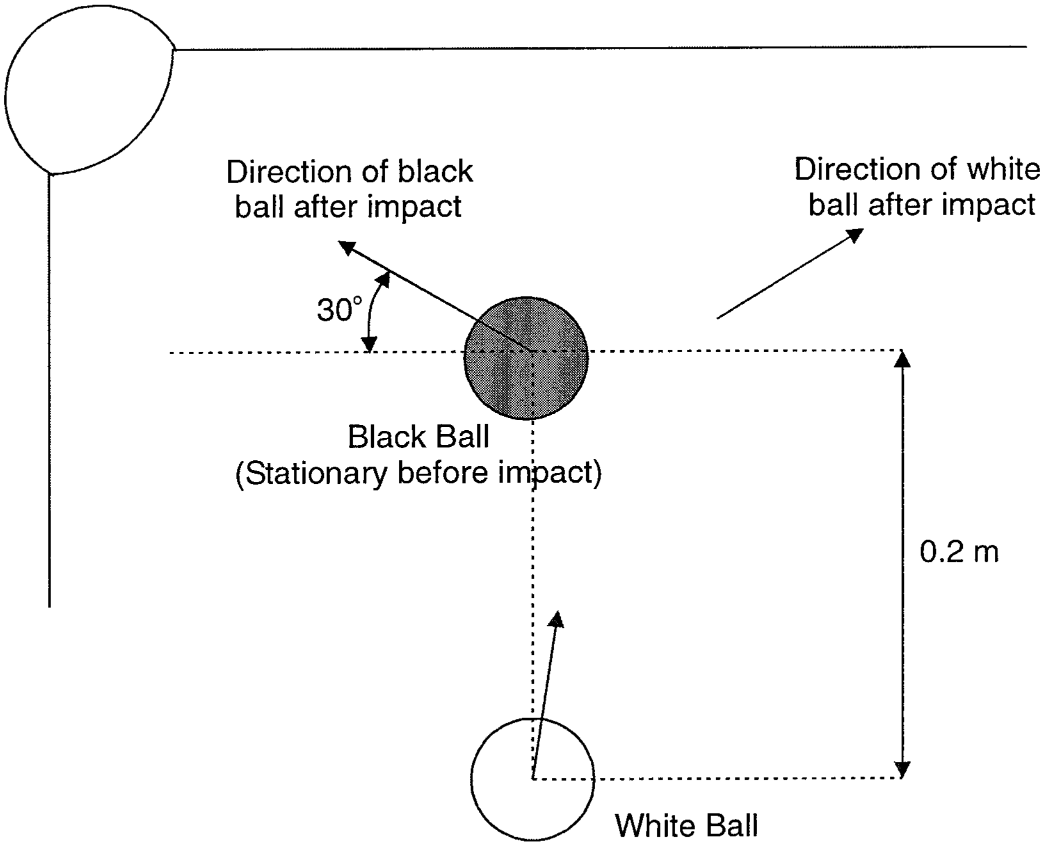
$m = 8.0$ kg
k (spring) = 2000 Nm^{-1}
The uncompressed length of the spring is 0.2 m.

What is the normal force exerted on the mass by the surface when it is at B? **(20 Marks)**

3) In a game of billiards, the white ball is to be collided into the stationary black ball such that after the impact the black ball is directed as shown in Figure 3.

Note: The white ball must hit the black ball directly and is not allowed a prior collision off of cushion.

Figure 3



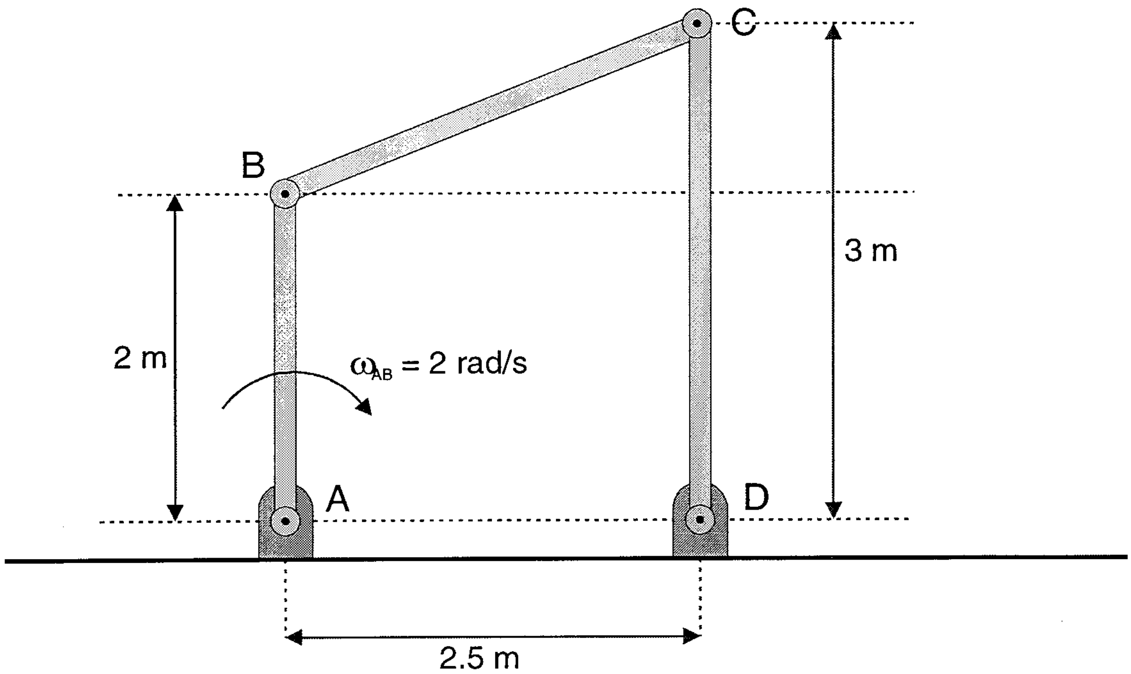
If the billiard balls are both 2.4cm in radius and equal in mass, and the coefficient of restitution for this impact is 0.9, in what direction does the white ball travel after the impact? **(24 Marks)**

The balls are smooth.

Hint: First determine the direction the white ball must be directed before the impact to send the black ball in the direction shown.

4) Link AB rotates at a constant angular velocity of 2.0 rad.s^{-1}. At the instant shown in Figure 4, what is the angular acceleration of the link BC? **(20 Marks)**

Figure 4



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Dynamics 2391 Formula Sheet

Rectangular Co-ordinates

$\boldsymbol{r} = x\hat{\boldsymbol{i}} + y\hat{\boldsymbol{j}}$ $\quad \boldsymbol{v} = \dot{x}\hat{\boldsymbol{i}} + \dot{y}\hat{\boldsymbol{j}}$ $\quad \boldsymbol{a} = \ddot{x}\hat{\boldsymbol{i}} + \ddot{y}\hat{\boldsymbol{j}}$ (Dots indicate time derivatives)

Polar Co-ordinates

$\boldsymbol{r} = r\hat{\boldsymbol{e}}_r$ $\quad \boldsymbol{v} = \dot{r}\hat{\boldsymbol{e}}_r + r\dot{\theta}\hat{\boldsymbol{e}}_\theta$ $\quad \boldsymbol{a} = (\ddot{r} - r\dot{\theta}^2)\hat{\boldsymbol{e}}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\boldsymbol{e}}_\theta$

Tangential and Normal Co-ordinates

$\boldsymbol{v} = v\hat{\boldsymbol{e}}_t$ $\quad \boldsymbol{a} = \dot{v}\hat{\boldsymbol{e}}_t + \dfrac{v^2}{\rho}\hat{\boldsymbol{e}}_n$

Radius of Curvature

$$\rho = \frac{\left(1 + y'^2\right)^{3/2}}{|y''|}, \text{ where } y' = \frac{dy}{dx} \text{ and } y'' = \frac{d^2y}{dx^2}$$

Relative Motion (in *non-rotating* co-ordinates)

$\boldsymbol{r}_B = \boldsymbol{r}_A + \boldsymbol{r}_{B/A}$, $\quad \boldsymbol{v}_B = \boldsymbol{v}_A + \boldsymbol{v}_{B/A}$, $\quad \boldsymbol{a}_B = \boldsymbol{a}_A + \boldsymbol{a}_{B/A}$

Transformation of a vector between 2 co-ordinate systems rotated relative to each other.

$\boldsymbol{b} = b_x\hat{\boldsymbol{i}} + b_y\hat{\boldsymbol{j}} = b_{\tilde{x}}\hat{\boldsymbol{i}}^{\sim} + b_{\tilde{y}}\hat{\boldsymbol{j}}_{\tilde{y}}$ If the $\hat{\boldsymbol{i}}^{\sim}, \hat{\boldsymbol{j}}^{\sim}$ unit vectors are rotated an angle θ anti-clockwise with respect to $\hat{\boldsymbol{i}}, \hat{\boldsymbol{j}}$ then

$$b_{\tilde{x}} = b_x\cos(\theta) + b_y\sin(\theta)$$

$$b_{\tilde{y}} = -b_x\sin(\theta) + b_y\cos(\theta)$$

Newton's Second Law for a system of N particles $\sum \boldsymbol{F}_i = m_T\, \boldsymbol{a}_G$
where m_T is the total mass of the system, $\boldsymbol{a}_G = \ddot{\boldsymbol{r}}_G$ is the acceleration of the centre of mass, $\boldsymbol{r}_G$, which is defined by $\boldsymbol{r}_G = \left(\sum_{i=1}^{N} m_i\boldsymbol{r}_i\right)\Big/ m_T$

Newton's Law of Universal Gravitation

$\boldsymbol{F} = \dfrac{Gm_1m_2}{r^2}$ where $G = 6.673\text{x}10^{-11}\ \text{m}^3\text{kg}^{-1}\text{s}^{-2}$

Central Force Motion

$r^2\dot{\theta} = \text{constant}$

Kinetic Energy of a particle of mass m and velocity v

$T = \dfrac{1}{2}mv^2$

Principle of Work/Energy

$\Delta T = U_{1\to2}$, where $U_{1\to2}$ is the work done by forces on a system between states 1 and 2.

$U_{1\to2} = \sum \int \boldsymbol{F} \bullet \mathrm{d}\boldsymbol{r}$. For a conservative force, $U_{1\to2} = -\Delta V$, where V is the potential energy of the force.

Potential Energy Functions

For a linear spring: $V = \frac{1}{2} k \delta^2$

For a mass in a uniform gravitational field: $V = mgy$

For a mass in a general gravitational field: $V = -\frac{GMm}{r}$

Power and Efficiency

$$Power = \frac{\mathrm{d}U}{\mathrm{d}t} = \boldsymbol{F} \bullet \boldsymbol{v}, \quad Efficiency = \eta = \frac{Output - Work}{Input - Work}$$

Principle of Impulse and Momentum

$$Impulse = \Delta \boldsymbol{p} = \int \boldsymbol{F}\, dt$$

Coefficient of Restitution

$$e = \frac{v'_{Bn} - v'_{An}}{v_{An} - v_{Bn}}$$

Relative Motion of 2 points on a Rigid body

For a polar co-ordinate system attached at A,

$$\boldsymbol{v}_B = \boldsymbol{v}_A + (\boldsymbol{v}_{B/A})_\theta = \boldsymbol{v}_A + r_{B/A}\omega_{BA}\hat{\boldsymbol{e}}_\theta$$

$$\boldsymbol{a}_B = \boldsymbol{a}_A + (\boldsymbol{a}_{B/A})_r + (\boldsymbol{a}_{B/A})_\theta = \boldsymbol{a}_A + (-r_{B/A}\omega_{BA}^2)\hat{\boldsymbol{e}}_r + (r_{B/A}\alpha_{BA})\hat{\boldsymbol{e}}_\theta$$

(Note: $\omega_{BA} = \omega_{AB}$ is the angular velocity of the link AB, and $\alpha_{BA} = \alpha_{AB}$ is the angular acceleration of the link AB)

Old maths formulae you may have forgotten

For the quadratic equation, $ax^2 + bx + c = 0$, the roots are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sine and Cosine Rules

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}, \qquad a^2 = b^2 + c^2 - 2ab\cos\theta$$